

Savings that Hurt: Production Rationalization and its Effect on Prices*

Mauricio Varela[†] Madhu Viswanathan[‡]

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Abstract

Production rationalization, the process of re-allocating production across facilities so as to reduce total costs, results in firms equating marginal costs across markets. This results in marginal costs, and hence prices, being higher in some markets and lower in others than otherwise would be without production rationalization. This paper proposes a model of competition that elicits these effects and the resulting consequences on consumer and producer surplus. The paper also presents empirical evidence on how production rationalization, in the form of fleet re-optimization, affected prices following the US Airways/American Airlines merger. Prices of the merged firm increased 10% on routes typically served by US Airways relative to routes typically served by American Airlines, and by 5% relative to US Airways' rivals' prices. Price-cost regressions confirm such price hikes were likely due to fleet re-optimization.

Keywords: production rationalization, merger effects, cost synergies, American Airlines

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[†]Corresponding author. Department of Economics, University of Arizona, Tucson, AZ 85721. E-mail: mvarela@email.arizona.edu

[‡]Department of Marketing, University of Arizona, Tucson, AZ 85721. E-mail: madhu@email.arizona.edu

1 Introduction

“The Agencies have found that (...) efficiencies resulting from shifting production among facilities formerly owned separately, which enable the merging firms to reduce the incremental cost of production, are more likely to be susceptible to verification and are less likely to result from anti-competitive reductions in output.” U.S. Department of Justice and Federal Trade Commission (1997)

Firms constantly re-allocate resources and production across facilities and technologies in response to changes in supply, demand, regulatory, and technological conditions. This process, known as production rationalization, can lead to dramatic changes in both the internal operations of the firm as well as market outcomes in general. Changes in trade deals, investments in infrastructure, expansions in input markets, all facilitate production rationalization by making it easier for firms to move inputs across facilities. For example, the North American Free Trade Agreement (NAFTA) has led firms to rationalize production across countries, changing the costs and the prices with which firms supply distinct markets. Aside from trade, another context in which production rationalization can have large effects is in post-merger integration, as merging parties re-optimize production across the newly merged entity.

Production rationalization can have positive consequences of some consumers and negative consequences for others. In industries where input costs are characterized by diseconomies of scale, firms may want to transfer inputs from facilities with low input costs to facilities with high input costs, lowering production costs at the latter. As these transfers increase how much input is acquired at the former facility, and as input costs have diseconomies of scale, these transfers result in production costs increasing at the former facility. As prices respond to production costs, prices decrease for consumers supplied by the latter facility and increasing for those supplied by the former. As some consumers gain and some lose, it becomes an empirical question to determine the benefit of policies that aim to facilitate or deter production rationalization, whether those policies be trade deals, merger approval, infrastructure investments, etc.

The empirical question on how consumers are affected by production rationalization should be given appropriate attention in merger analysis. The FTC’s Horizontal Mergers Guidelines state how, in their reviews, the Agencies (the FTC and the DOJ) give positive credit to efficiencies that are merger-specific and likely to result in reductions of incremental costs. They explicitly state that *“shifting production among facilities formerly owned separately”* are more likely to be credited than other forms of efficiencies. We suggest caution when analyzing such efficiencies, as production rationalization may result in higher incremental costs for some consumers, even though it results in lower incremental costs for others. Such caution is also to be suggested for potential merging parties, so as to not ignore how savings in one market can be offset with higher costs in other markets.¹

¹It is incorrect to assume production rationalization is always good for the rationalizing firms. The ability to rationalize production implies the inability to commit certain cost structures to certain markets. A firm facing a high cost rival in one market and a low cost rival in another market may prefer to have low cost of supplying the former market and a high cost of supplying the latter market instead of having a medium cost of supplying both markets.

In this paper we develop a model of competition that formalizes the effects of production rationalization. We then show how, due to the US Airways - American Airlines merger, production rationalization in the form of fleet re-optimization could have increased prices for some customers without significantly decreasing prices for others. Finally, we test our intuition using price and cost data for the US domestic airline industry.

Why would have fleet re-optimization resulted in higher prices for some without resulting in lower prices for others? Prior to their merger, US Airways utilized for domestic operations a fleet of fuel efficient Airbus A320s. In contrast, American Airlines utilized the gas-guzzling McDonnell Douglas MD80s. If US Airways had some slack in the operation of its A320s, the merger would have allowed American Airlines to tap into US Airways' underutilized fleet, saving costs for the merged firm. However, due to the difference in scale, all slack in the A320s would have gone to service inframarginal consumers. American Airlines' marginal consumer would continue to be serviced with MD80s. US Airways marginal consumer would no longer be serviced with A320s, and would be serviced instead with MD80s. The marginal cost of servicing a US Airways customer would have increased, while the marginal cost of servicing an American Airlines customer would have remained the same. As prices are determined by the *marginal* consumer, prices for US Airways' customers would have risen while prices for American Airlines' customers would have remained flat.

We use a Difference-in-Difference estimator to test for such price changes. We find prices of the merged firm (i.e. AA/US) increased 10% more on predominantly US Airways' routes² than on predominantly American Airlines' routes. In addition, merging parties' prices increased at least 5% more than rivals' prices on predominantly US Airways' routes. We take this as strong evidence that the merger did indeed result in prices increasing for US Airways customers. To show that such price increases were due to production rationalization, we correlate operating costs of MD80s and A320s with prices and find prices of the merging parties on predominantly US Airways' routes track the costs of the A320 more closely than those of the MD80 prior to the merger, and vice-verse after the merger. In contrast, prices on predominantly American Airlines' routes track the costs of the MD80 more closely than those of the A320 both pre- and post- merger. We take this as evidence that production rationalization, in the form of fleet re-optimization of A320s and MD80s, is likely to have caused price increases on US Airways' operations and no price effects on American Airlines' operations.

This paper touches on both merger efficiencies and multi-market competition. Most literature on merger efficiencies³ focuses on the type and size of efficiencies that mitigate consumer welfare loss from reductions in competition. The literature on multi-market competition focuses on how cross-market supply spillovers (e.g. Bulow et al. (1985)) or conduct spillovers (e.g. Bernheim and Whinston (1990)) affect market outcome, with empirical work (cite in footnote) focusing mainly on showing

²A *predominantly US Airways' route* is a route that, in the year prior to the merger, US Airways' share was larger than the average share, and the Herfindahl index's expected change due to the merger was less than 100 points.

³See, for example, Farrell and Shapiro (1990) for the size of marginal cost efficiencies; Werden and Froeb (1998) on how merger efficiencies constrain future entry; Stennek (2001) on information transfer as a form of efficiencies.

how increases in multi-market contact increased collusive behavior between competitors. Our paper provides an alternate mechanism (production rationalization) to explain increases in prices that does not rely on competition. In particular, this paper argues how a merger generates the opportunity for rationalizing production across markets. Even if there are no reductions in competition, this rationalization has effects on the marginal cost of production in every market, altering prices, rivals' responses, and consumer welfare. [[other literature on dis-economies of scope and multi-plant operations??]]

The paper also adds to the growing empirical literature that evaluates merger effects *post facto*, and particularly in the airline industry.⁴ Peters (2006) shows how price changes for five different airline mergers were mostly due to changes in unobserved costs (inferred from pricing decisions), and not so much as changes in market structure. Werden et al. (1991) specifically compares price changes following the Northwest-Republic and TWA-Ozark mergers across routes that suffered loss of competition relative to those that did not. They find prices increased on select routes that did not suffer loss of competition. Unfortunately, as the main focus of their paper were routes that did suffer loss of competition, the authors do not explore further plausible causes for such price increases. Our paper provides one possible explanation for their findings.

Finally, the airline industry has been vastly used to illustrate various cross-market effects, including multi-market contact (e.g. Evans and Kessides (1994) and Ciliberto and Williams (2012)), density economies (e.g. Brueckner and Spiller (1994)), hub dominance (e.g. Borenstein (1989)), and endogenous network formation (e.g. Bamberger and Carlton (1996)). We add to this literature by detailing how production rationalization affects prices.

In the following section we propose a simple theoretical model that formalizes how a multi-facility firm equates marginal cost across production facilities and how allowing for such production rationalization affects costs and prices. Section 3 explores evidence of price changes in the US airline industry following the American Airlines - US Airways merger and relates such price changes to the aforementioned theoretical model. Section 4 discusses briefly the implications for policy and firm behavior. Conclusions follow.

2 Production Rationalization and its Effect on Prices and Welfare

2.1 A Simplified Model of Competition

Assume two distinct markets, A and B , each with I_m firms, $m \in \{A, B\}$. Firms face a smooth, downward sloping, (weakly) concave, inverse⁵ demand in each market that is a function of total

⁴Articles that investigate, within the airline industry, price changes due to lost competitors include Luo (2014), Goolsbee and Syverson (2008), Kwoka and Schumilkina (2010), Gayle (2008), and Kim and Singal (1993).

⁵A similar analysis can be constructed using demand curves and having firms choose both prices and output simultaneously, but such analysis would have to specify rationing rules (as in Kreps and Scheinkman (1983) and Davidson and Deneckere (1986)). To avoid the complications that arise from defining alternative rationing rules, we chose to model the game in strategic substitutes.

market output in such market: $P_m(Q_m)$ where $Q_m \equiv \sum_{i \in I_m} q_{im}$ and q_{im} is firm i 's output in market m . In what follows, denote with Q_{-im} the aggregate sales in market m of all firms but firm i . Also denote with $R_m(q_i, Q_{-i})$ the revenue of firm i (active in market m) given own sales q_i and rival sales Q_{-i} : $R_m(q_i, Q_{-i}) \equiv P_m(q_i + Q_{-i})q_i$.

Firms utilize two distinct inputs in the production of goods, τ and φ , such that the firm i 's production is governed by $q_i = \vartheta^i(\tau_i, \varphi_i)$. $\vartheta^i(\cdot, \cdot)$ is twice differentiable, strictly increasing in τ , weakly increasing in φ , weakly jointly-concave, and presents (weak) increasing differences: $\frac{\partial^2 \vartheta^i}{\partial \tau \partial \varphi} \geq 0$.⁶ The costs of τ and of φ are given by $C^i(\tau, \varphi)$, with positive marginal costs for both inputs, strict diseconomies of scale in τ and weak diseconomies of scale in φ . Using subscripts to denote partial derivatives (e.g. $C_\tau^i \equiv \partial C^i / \partial \tau$), the assumptions on costs are expressed as: $(C_\tau^i, C_\varphi^i, C_{\tau\tau}^i) > 0$ and $C_{\varphi\varphi}^i \geq 0$.

When production rationalization is infeasible, each firm simultaneously chooses input vectors (τ, φ) so as to maximize profits in their own markets:

$$\begin{aligned} (\tau_i^*, \varphi_i^*) &= \arg \max_{\tau, \varphi \geq 0} R_m(\vartheta^i(\tau, \varphi), Q_{-i}^*) - C^i(\tau, \varphi) \\ Q_{-i}^* &= \sum_{j \neq i} \vartheta^j(\tau_j^*, \varphi_j^*) \end{aligned} \quad (1)$$

Prior to characterizing the equilibria of this game, we introduce the ‘adjusted’ marginal cost of τ : the additional cost of τ required to increase a marginal unit of output: $C_\tau^i / \vartheta_\tau^i$. We call this the ‘adjusted’ marginal cost of τ .

The following Lemma describes how, in equilibrium, firms equate marginal revenue to marginal cost, as expected.

Lemma 1. *When production rationalization is infeasible, in any equilibria of the above game, firms with positive sales equate marginal revenue to the ‘adjusted’ marginal cost of input τ : i.e. $R'_m = C_\tau^i / \vartheta_\tau^i$.*

Production Rationalization Assume now that firm a , located in market A , can rationalize production with firm b , located in market B . Specifically, assume b 's input units τ_b are freely exchangeable with a 's input units τ_a on a one-to-one basis. When choosing inputs, firm a simultaneously makes an offer to firm b of a fixed transfer payment T in exchange for t units of τ_b . These input units can then be⁷ used by firm a to complement its own τ_a input. Both T and t can be negative, equivalent to a selling to b output units. Given this, how do market outcomes change relative to when production rationalization is infeasible?

⁶Concavity and increasing differences guarantee, along with the other assumptions on inverse demand and costs, that each firm's optimization problem is concave.

⁷Results are qualitatively similar for costly transfers of inputs, as long as such transfers are not too expensive: less than the difference in the two firms' pre-merger marginal costs.

To avoid triviality, we assume both firms are active in their respective markets absent production rationalization:

Assumption 1. *When production rationalization is infeasible, in equilibrium firms a and b sell positive amounts in their respective markets: $\vartheta^a(\tau_a^*, \varphi_a^*) > 0$ and $\vartheta^b(\tau_b^*, \varphi_b^*) > 0$.*

Given how general the contracting structure is, in equilibrium, firms a and b choose input allocations that maximize joint profits (cf. Dixit (1983)), avoiding issues of double marginalization. Hence, equilibrium outcomes are characterized by firm a and b 's joint problem:

$$\begin{aligned} \max_{\tau_a, \tau_b, t, \varphi_a, \varphi_b} & R_A(\vartheta^a(\tau_a + t, \varphi_a), Q_{-a}^*) + R_B(\vartheta^b(\tau_b, \varphi_b), Q_{-b}^*) - C^a(\tau_a, \varphi_a) - C^b(\tau_b + t, \varphi_b) \quad (2) \\ \text{s.t.} & \tau_m + t \geq 0 \quad \tau_m \geq 0 \quad \varphi_m \geq 0 \quad \forall m \in \{a, b\} \end{aligned}$$

and by eq. 1 for all other firms.

Proposition 1 develops the comparative statics used to compare equilibrium outcomes with and without production rationalization. In essence, production rationalization allows the rationalizing firms to reshuffle production from high-cost facilities to low-cost facilities, increasing profits. The effect on consumers and rivals is mixed, with some gaining and some losing: by reshuffling inputs, the rationalizing firms (weakly) increases marginal cost in markets that used to be served by the low-cost inputs. This increase in marginal cost results in the rationalizing firm decreasing sales in that market, rival firms increasing sales, and a net decrease in total sales. By extension, consumer welfare decreases in these markets. The opposite occurs in markets served by the high-cost input. The net effect on consumers can be negative if consumers in the market with the low-cost inputs are affected more than consumers of the other markets.

Proposition 1. *If, without production rationalization, firm a 's equilibrium marginal cost is higher than firm b 's, then with production rationalization:*

1. *Firm a 's sales (weakly) increase and firm b 's (weakly) decrease.*
2. *Total sales in market A (weakly) increase and total sales in market B (weakly) decrease.*
3. *Consumer surplus (weakly) increases in market A and (weakly) decreases in market B .*

The opposite effects occur if, without production rationalization, firm a 's equilibrium marginal cost is lower than firm b 's.

In the proof, provided in the appendix, one notes that, with production rationalization, the rationalizing firms' 'adjusted' marginal cost value is in between the 'adjusted' marginal cost values the firms would have had without production rationalization. As a result, production rationalization results in the 'adjusted' marginal cost increasing for a subset of consumers, and prices with it. Although the 'adjusted' marginal cost of serving the remaining consumers may have decreased, if price

shifts on the former set of consumers are sufficiently significant, the net effect on consumer welfare is negative. If, on the other hand, the price decrease on the later set of consumers is sufficiently significant, production rationalization results in a net consumer welfare gain. Hence, the effect of increased production rationalization on consumer welfare is an empirical question.

The proof also shows how, if sales of the rationalizing firms are positive in both markets, marginal revenue must be equal across these two markets. If production is positive in both markets, ‘adjusted’ marginal costs also must be equal across both markets. The intuition is straightforward: if the rationalizing firms wants to increase sales in a given market, instead of purchasing additional inputs, it can always reduce sales in the alternative market and transfer the saved inputs to the former market. As such, marginal revenue in the alternative market serves as an opportunity cost of increasing sales in the focal market. As for why the firm equates ‘adjusted’ marginal costs across markets, if this were not so, the firm could always acquire less inputs in the expensive market, acquire more inputs in the inexpensive market, and transfer those additional inputs across markets. This would keep total sales constant but decrease input costs.

Key assumptions that drive Proposition 1 are (dis)economies of scale in inputs, cheap transfer of goods across markets, and the exogenous market structures. The validity of these assumptions will depend on the specific setting under study. Diseconomies of scale in inputs are very common in manufacturing, driven by capacity constraints and/or scarcity (e.g. limestone, used in cement manufacturing, does not travel far and can be scarce depending on quarry reserves). The more relevant assumption is the cheap transfer of inputs *and* the exogenous market structure (i.e. large barriers to entry), such that firm a may want to sell inputs to firm b , but may not want to makes sales in market B directly. Such situation may be common if there are market-specific fixed costs each firm needs to incur to participate in a given market: e.g. store fronts, advertising, distribution channels, branding, etc. In such situations, market B may not be large to accommodate firm a in addition to all current players, despite a having a beneficial cost structure. It could also be that the complementary inputs required to make sales (φ in the model above) are not available to firms foreign to the market: e.g. a tire manufacturer that sells tires directly to consumers (market B) cannot sell cars (market A) because it does not have access to other inputs required for cars (e.g. design, dealerships, assembly plants, etc.).

The assumption that an *input* is the transferable good is not critical. The model allows $\vartheta(\tau, \varphi) = \tau$, in which case it is the final output that has diseconomies of scale and that is being rationalized across markets.

[[Add proposition that production rationalization is not always optimal for rationalizing firms: they lose the ability to commit certain cost structures to certain markets]]

2.2 Production Rationalization and Mergers: An Illustrative Example

Prior to their merger, US Airways operated a more fuel efficient fleet than American Airlines. However, due to the narrower scope of their operation, it is likely that US Airways’ fleet was

underutilized relative to how American Airlines could have utilized such fleet. Hence, the merger provided an opportunity to optimize fleets in a way that increased the utilization of US Airways' fuel efficient fleet.⁸ This reshuffling could have lowered costs for the merging firms and increased profitability.⁹ However, such fleet re-optimization may have also increased prices for US Airways customers. The following example illustrates how such price hikes may have arisen, quantifies how much consumers could have been hurt from such price hikes, and provides context to understand the empirical exercises that follow. These empirical exercises do not quantify consumer welfare loss as such exercise would require strong assumptions on pricing decisions, cost estimates, and rival firm repositioning. Such exercise is out of the scope of this paper, which is why we provide the following illustrative example.

Consider a setting with two firms, American Airlines (AA) and US Airways (US), each operating in distinct markets, each facing a single competitor. For simplicity, let Southwest (WN) be US Airways' competitor and Delta (DL) be American Airlines' competitor. Let demand in US Airways' market be symmetric for both US Airways and Southwest, given by: $D_i(p_i, p_j) = \alpha - p_i + \gamma p_j$, for (i, j) representing US Airways and Southwest, respectively, or vice-versa. Demand in the American Airlines' market is similar to that in US Airways market, differing solely by scale: $D_i(p_i, p_j) = \tau(\alpha - p_i + \gamma p_j)$ for (i, j) representing (AA, DL) or vice-versa. US Airways, Southwest, and Delta all have a common cost: c . American Airlines has a higher cost: $c_{AA} = c + \Delta$, in accordance with American Airlines having the more expensive fleet. Additionally, we assume US Airways has a capacity limit κ , large enough such that it poses no restriction prior to the merger. It is through this capacity limit that diseconomies of scale enter the example.¹⁰

Prior to the merger, each firm maximizes profits independently in each market. Equilibrium prices and quantities are (Southwest's values are identical to US Airways'):

$$p_{US}^* = \frac{\alpha + c}{2 - \gamma} \quad p_{AA}^* = \frac{\alpha + c}{2 - \gamma} + \Delta \frac{2}{4 - \gamma^2} \quad p_{DL}^* = \frac{\alpha + c}{2 - \gamma} + \Delta \frac{\gamma}{4 - \gamma^2}$$

⁸Aircraft utilization can differ across firms and across routes due to various reasons: turnaround times at airports, propensity for delays (e.g. weather, mechanical, or congestion), and differences in flight distances. To the extent that the differences in utilization are induced by exogenous factors (e.g. weather, distance, etc), carriers can optimize profitability by assigning the cost-efficient aircraft to the markets with highest utilization: e.g. AA-US can use the inefficient MD80s on US' lesser utilized routes and utilize the A320s on AA's more utilized routes. As long as the source of underutilization is not tied to the aircraft (e.g. congestion, weather, and distance are all irrespective of aircraft type), profits can be improved by allocating aircraft optimally.

⁹There are very reasons why such profit gain may be substantial but could not have been achieved without the merger. For example, barriers to entry into certain airports (e.g. Berry (1992); Boguslaski et al. (2004); Goolsbee and Syverson (2008)) and/or network optimality considerations (e.g. Oum et al. (1995); Lohatepanont and Barnhart (2004)) could have barred US Airways from entering American Airlines markets prior to the merger. Also, aircraft swaps between US Airways and American Airlines are also likely to have been infeasible due to the need to also swap crews and maintenance operations along with the aircraft. Finally, the market for outsourcing aircraft operations to third party providers is relatively small, possibly due to incomplete contracting considerations, suggesting that it would have been infeasible for the merging firms to create a third party service provider that optimized fleet utilization across both carriers.

¹⁰Although a capacity constraint is not entirely consistent with a smooth cost function (as assumed in section 2.1), this example could be approximated arbitrarily close with a smooth, but very convex, cost function.

$$q_{US}^* = \frac{\alpha - (1 - \gamma)c}{2 - \gamma} \quad q_{AA}^* = \tau \left[\frac{\alpha - (1 - \gamma)c}{2 - \gamma} - \Delta \frac{2 - \gamma^2}{4 - \gamma^2} \right] \quad q_{DL}^* = \tau \left[\frac{\alpha - (1 - \gamma)c}{2 - \gamma} + \Delta \frac{\gamma}{4 - \gamma^2} \right]$$

To ground the example to industry data, note that 2014 operating costs for the MD80 and MD90 series were, on average, 12.0 ¢/seat-mi. In contrast, operating costs for the A320 and B737-300 were 9.8 ¢/seat-mi.¹¹ Hence, let c be 9.8 and Δ be 2.2. The industry elasticity, according to InterVISTAS (2007), is of -1.4 and we assume an operating margin of 20%.¹² The operating margin and the industry elasticity identify α, γ , who take on the values 4.17 and 0.86, respectively. The capacity parameter κ is calibrated using US Airways' 2014 utilization rate of its A320 fleet, of 73%. This value is the ratio of US Airways' A320s' "passenger-miles by available aircraft days" relative to the 75th percentile value of rival firms A320s' value. As q_{US}^* is 2.45, κ is 3.36. Finally, τ is obtained by setting q_{AA}^*/q_{US}^* equal to American Airlines' 2014 domestic passenger-seat-miles divided by those of US Airways. Given this ratio is 1.33, τ is 2.04.

Post merger, American Airlines and US Airways' maximize profits jointly, where the only linkage comes through the capacity constraint:

$$\begin{aligned} \max_{(p_a, p_b, q_a, q_b) \geq 0} \quad & D_{AA}(p_a, p_{DL})p_a + D_{US}(p_b, p_{WN})p_b - cq_a - (c + \Delta)q_b \\ \text{s.t.} \quad & q_a + q_b \geq D_{AA}(p_a, p_{DL}) + D_{US}(p_b, p_{WN}) \quad ; \quad q_a \leq \kappa \end{aligned} \quad (3)$$

This linkage allows American Airlines to tap into the underutilized capacity at US Airways. As described in section 2.1, the joint firm equates marginal revenue across markets. The lower-cost capacity is fully utilized and marginal cost is driven by the higher cost capacity. Prices do not change in American Airlines' market but do rise in US Airways' market, up to the same levels as in AA's market. Table 1 summarizes prices, quantities, and profits.

As prices are unaltered in market B , consumer surplus there is unaffected by the merger. However, prices increased in market A . This price increase induces a consumer welfare loss of 4.32, calculated as the line integral of the sum of each carrier's demand with respect to the price change. This consumer welfare loss is equivalent to 10% of market A 's pre-merger consumer welfare and 36% of market A 's pre-merger profits. Profits for the merging firm, however, increased by 3.99, a 35% increase over the joint-firm's pre-merger profits. These are very large effects.

¹¹Operating costs per aircraft type were obtained from the Bureau of Transportation Statistics' Financial Data (P.5.2 Schedule), from which a cost per hour was calculated. Using the BTS' T-100 Segment data, we obtained available seat-miles and total hours of operation, by aircraft type. Merging these two data sets we obtained an operating cost per available seat mile. As the P.5.2 Schedule data did not include selling costs, we added to the estimates an additional 2 cents per available seat mile, the average selling costs in the industry per 10-k reports.

¹²Southwest's average EBITDA for 2013-2015 was 26%. AMR's was, for the same time period and excluding regional operations, 17%.

Table 1: Equilibrium Prices, Quantities and Profits for the Illustrative Example

Mkt	Carrier	Pre-merger			Post-Merger		
		p^*	q^*	π^*	p^*	q^*	π^*
A	US	12.25	2.45	6.00	13.60	1.60	N/A
	WN	12.25	2.45	6.00	12.83	3.03	9.18
B	AA	13.60	3.26	5.21	13.60	3.26	15.2
	DL	12.83	6.17	18.7	12.83	6.17	18.7

Prices are in cents per seat-mile. Quantity is shown per-unit of market size. This is a hypothetical example and neither companies nor values are representative of actual behavior

Of critical note, this example shows how a merger that has *no* unilateral effects and has *substantial, merger-specific, variable* cost efficiencies, efficiencies that could be labelled as cognizable efficiencies under the FTC Horizontal Merger Guidelines, can result in substantial consumer welfare *loss*. Ignoring the role of production rationalization and its subsequent impact on prices as presented here, this merger would receive little scrutiny from regulators as horizontal effects are unlikely and the merger presents significant cognizable efficiencies: the merging parties could credibly suggest that, by increasing the utilization of US Airways’ efficient fleet, American Airlines could reduce operating costs by 5.1%¹³, and these savings would likely be passed on to consumers as they are on variable costs. Important to note, it is *marginal* cost that affects pricing decisions, not *variable* costs.

In what follows we document evidence that the US Airways - American Airlines merger did indeed result in price increases for US Airways customers.

3 US Airways - American Airlines Merger Effect on Prices

3.1 A Brief on the US Airways - American Airlines Merger

US Airways was a major American airline that merged with American Airlines in December of 2013. Initially announced in February 2013, the merger allowed American Airlines to emerge from bankruptcy. The Department of Justice did issue a complaint against the merger on the grounds that it would facilitate coordinated effects among industry players and reduce competition at select airports (e.g. Washington Reagan, LaGuardia, etc.). However, the DOJ reached a settlement with the merging parties in November 2013 in which the merging parties would divest landing slots and gates at select airports as a way of decreasing barriers to entry for low cost carriers (cf. United States District Court for the District of Columbia (2014)). The merging parties finalized consolidating operations in April 2015, when they obtained a Single Operating Certificate from the FAA.

Prior to the merger, US Airways’ domestic fleet consisted of more fuel efficient aircraft than American

¹³Cost savings would be calculated as $(\kappa - q_{US}^{pre}) \Delta = 1.99$, and operating expenses would be $c_{AA} \cdot q_{AA} = 39$.

Airlines'. Specifically, US Airways' mainline, narrow-body, fleet consisted of 93 Airbus A319s, 75 A321s, 72 A320s, and 56 other aircraft, with an average age of 11 years. In contrast, American Airlines' mainline, narrow-body, fleet consisted of 195 Boeing 737s, 191 McDonnell Douglas MD-80s, and 106 Boeing 757s, with an average age of 14 years.¹⁴ The merging parties did view the merger as a way of optimizing fleets, calling for the "Right Aircraft in the Right Place at the Right Time [*sic*]" (AMR Corporation and US Airways Group, Inc (2013)) creating \$550M in cost synergies, a 21% increase over the combined firms' 2013 operating profits (cf. American Airlines Group, Inc (2014)).

We interpret the above synergies to imply the joint-firm would be able to increase utilization of less expensive aircraft and decrease utilization of expensive aircraft. It is likely that such fleet re-optimizing changed the *marginal* aircraft for US Airways operations from a low-cost aircraft (e.g. A320) to a high-cost aircraft (e.g. MD80). This increase in marginal cost would be likely reflected in an increase in price. The following subsection explores this hypothesis.

3.2 Data

Sources We use data from the Bureau of Transportation Statistic's (BTS) Airline Origin and Destination Survey (DB1B database), which contains a 10% sample of all itineraries sold for domestic flights in the US. The data is reported quarterly and we obtain data from the first quarter of 2010 up to the second quarter of 2016. We aggregate up the itineraries to the carrier-route-quarter level, and define a route as a unidirectional city pair.¹⁵ The unit of observation is a carrier-route-quarter triplet: e.g. American Airlines' service between Tucson and Miami on the third quarter of 2014. The data contains information on passengers served, the average price¹⁶ paid per passenger, and the flight structure: non-stop or connecting.

We additionally use the Air Carrier Statistics database (T-100 Segment). This data contains, at the monthly level, the number of seats assigned and passengers flown by each carrier's aircraft class on each (directional) airport pair. An aircraft class is a manufacturer-model pair: e.g. 737-300, 737-800, A320-100/200, etc.; which we categorize into one of three categories: regional jets, mainline jets, and other aircraft.¹⁷ The Air Carrier Statistics data is aggregated to a carrier-route-quarter and

¹⁴US Airways Corp, including US Express, also operated 26 twin-aisle aircraft, 67 regional jets, and 44 turbo-props. AMR, including American Eagle, also operated 122 twin-aisle aircraft, 245 regional jets, and 9 turbo-props. Cf. US Airways Group, Inc. (2013) and AMR Corporation (2013))

¹⁵As a carrier can service a route through multiple flight structures (e.g. non-stop flight, one-stop flight connecting in X hub, one-stop flight connecting in Y hub, two-stop flights, etc.), we aggregate only the itineraries that were serviced with the modal flight structure. We do, however, take total sales regardless of flight structures when calculating shares. Also, we drop the observations, i.e. carrier-route-quarters, in which the modal flight structures involved two or more connections.

¹⁶We drop observations for which average prices are outrageous—below \$25 dollars or above \$2,500 dollars.

¹⁷Classified as a mainline jet are all of Airbus's A319, A320, A321, and A330 class jets, all of Boeing's 737, 757, and 767 class jets, as well as the MD80/90 family. Regional jets are all of Embraer's and Bombardier's jet aircraft and Boeing's 717. Other aircraft is any other aircraft not mentioned here.

merged onto the DB1B data.¹⁸ In doing so we record the modal aircraft category used to service each leg of each route and the average load factor on those legs. As connecting service has two flight legs, we retain the maximum load-factor across the two legs.

Lastly, we incorporate cost information using the BTS' Form 41 - Financial Data, Schedule P.5.2. This data reports, at the quarterly level, domestic operating costs and statistics for each carrier and for each aircraft class. From this data we retain operating costs per hour of airtime, by quarter, carrier, and aircraft class, where operating costs include costs of crews, fuel, maintenance, depreciation, and rental equipment. As the T-100 Segment data has information on air time and on flying distance, we merge the Schedule P.5.2 cost data with the T-100 data and obtain quarterly operating costs per passenger-seat mile, for each carrier and each aircraft class.

As the DB1B is an extensive data set with many charter carriers, executive jets, and freight carriers, we drop all carriers with less than 1% national market share, calculated yearly. We drop small routes, routes to small cities, routes within Alaska or Hawaii, and routes to, from, or within US territories.¹⁹ We consider a carrier to be active on a route at a certain quarter if the carrier services at least a thousand passengers or has at least 15% market share in that quarter.²⁰

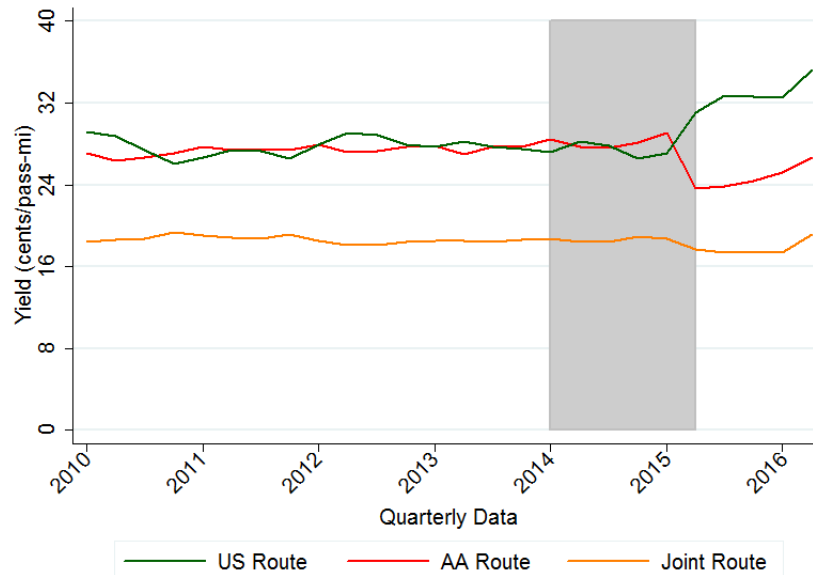
Route Classification In the analysis that follows, we would like to distinguish how the American Airlines / US Airways merger affected prices for US Airways relative to American Airlines. However, in the years that follow the merger US Airways ceases to exist and its operations are taken over by American Airlines. So as to have a 'hold' on how prices of the merged firm changed on its US Airways operations, we classify routes according to which of the two merging parties had a more dominant position on that route prior to the merger. Specifically, we use 2013 yearly sales to calculate the Herfindahl index for each route, and the projected change in such index that a merger between US Airways and American Airlines would have implied. Any route in which this projected change is larger than 100 points is classified as a *Joint* route. For the remaining routes, any route in which US Airways' share is larger than the average share (i.e. $s_{US} > 1/N$) and American Airlines' share is less than the average share is classified as a *US Airways* route. Conversely, routes in which American Airlines' share is larger than the average share and US Airways' share is smaller than the average share is an *American Airlines* route. Routes in which neither carrier had shares larger than the average share is classified as a *Non-Served* route. With this route classification we are able to follow how the merging firms' prices changed after the merger on routes that were typical US Airways routes (e.g. Phoenix-Vegas), compared to prices on routes that were typical American Airlines routes (e.g. Chicago-Miami).

¹⁸For each itinerary coupon, the DB1B data reports two carriers: an *operating carrier*, responsible for operating the flight, and a *ticketing carrier*, responsible for sales of the ticket. The DB1B is aggregated to the carrier-route-quarter level utilizing the *ticketing carrier*. However, the identity of the modal *operating carrier* is retained, and it is on this modal carrier with which the match with the T-100 data is made.

¹⁹Small routes are those with less than 100 quarterly passengers. Small cities are those with less than 100 daily enplanements.

²⁰1,000 passengers over a quarter is equivalent to 80 weekly passengers, barely enough to justify a single weekly flight on a regional jet. Note that we do take these offerings into account when calculating market share.

Figure 1: AA/US Yields on *US Airways*' routes, *American Airlines*' routes, and *Joint* routes



US Airways' routes are those in which US Airways had a dominant position in 2013 and American Airlines did not. American Airlines routes are those in which American Airlines had a dominant position in 2013 and US Airways did not. Joint routes are those in which both carriers had a dominant position in 2013. See text for details on the exact constructs.

Measures of Price We use two different measures of 'price'. The first, yield, is the average revenue per passenger-mile. It is commonly used in the industry and accounts for how longer routes cost more. The second, log of the average price per passenger, is used in other academic papers and allows for a straightforward interpretation of elasticities and semi-elasticities.

AA/US Prices We refer to the merging firms' prices as *AA/US* prices, after the carriers IATA abbreviation. When referring to *AA/US* prices on *US Airways* routes, however, we refer solely to US Airways' prices pre-merger, and American Airlines' prices post-merger. Although American Airlines' may have had, on these routes, some sales pre-merger, American Airlines' prices are excluded. Similarly, when referring to *AA/US* prices on *American Airlines* routes, US Airways' pre-merger prices are excluded. This strategy removes potential noise from sporadic service when analyzing *AA/US* prices. For *Joint* routes, pre-merger prices of both carriers are retained.

3.3 US Airway Prices vs American Airlines Prices

In this first analysis we explore if the merging firms' prices on *US Airways* routes increased after the merger relative to prices on *American Airlines* routes. Figure 1 shows yield of the merging parties over time, by route type. So as to reduce noise from seasonality variation and persistent carrier differences, shown in figure 1 are the residuals of a regression of yield on quarter and carrier fixed effects, centered at average values. The grayed area shows the merging period (2014Q1 - 2015Q1).

As is apparent from the figure, yields on *US Airways'* routes increased significantly after the merger relative to yields on American Airline routes.

We formalize the above graph using a difference-in-difference estimation. The first difference compares prices before and after the merger. The second difference compares prices across route types: *Joint* routes (J), *US Airways'* routes (U), and *American Airlines'* routes (omitted category). All *Non-Served* routes are excluded from the estimation sample, as neither carrier was a competitive player on those routes the year prior to the merger and, hence, it is uncertain how they are pricing those routes, neither before nor after the merger. The merging period is also excluded from the sample, as it is unclear to what extent the merging parties were coordinating operations during this time.

The exact empirical specification is

$$yield_{irt} = \beta^U d_r^U d_t^{\text{pst}} + \beta^J d_r^J d_t^{\text{pst}} + \alpha^U d_r^U + \alpha^J d_r^J + \alpha^{\text{pst}} d_t^{\text{pst}} + \alpha^x x_{irt}^{(1)} + \epsilon_{irt} \quad (4)$$

where $yield_{irt}$ is carrier i 's yield, American Airlines' or US Airways', on route r in quarter t . d_r^U and d_r^J are dummy variables indicating *US Airways* routes and *Joint* routes, respectively. d_t^{pst} is a time dummy for all quarters after the merger: 2015Q2 and onward. $x_{irt}^{(1)}$ are a set of controls for costs and market power: a dummy for non-stop service, a dummy of US Airways' prices on joint routes, a dummy for American Airlines' prices after declaring bankruptcy (2012Q1), load-factor, the Herfindahl index on the route, the number of carriers, of non-stop carriers, of low-cost carriers, and of potential entrants;²¹ market share at endpoint cities –averaged across both cities–, and the number of non-stop routes the carrier services at the endpoint cities –averaged across both cities–.

A positive value on β^U is indicative that prices of the merged firm increased on *US Airways* routes relative to *American Airlines* routes following the merger. Table 2 shows the results from this estimation. Depending on the specification, following the merger yields on US Airways routes increased between 0.7 and 4.9 ¢/passenger-mile more than what yields on American Airlines routes increased. The results are statistically significant across specifications II-V, which control for long-run differences across routes and changes in competition within routes. They are also large: given an average yield of 26 ¢/passenger-mile, a price increase of 2.5 ¢/passenger-mi (specification III) is almost a 10% increase, twice the 5% SSNIP benchmark used in antitrust to flag a merger as potentially harmful to consumers (cf. U.S. Department of Justice and Federal Trade Commission (1997), section 4.1.2). Specification (V) uses log-price as the dependent variable and there too does the merger appear to induce a 12% increase in prices on *US Airways'* routes over *American Airlines'* routes.

The estimates for *Joint* routes appear to reflect a positive, but small, price increase following the merger. One would have expected prices to increase on these routes as the merger increased the

²¹Low cost carriers are all carriers excluding United Airlines, Delta Airlines, Continental Airlines, and Alaska Airlines. Potential entrants are defined as carriers with flights at both end-points of a route but no service on the route itself.

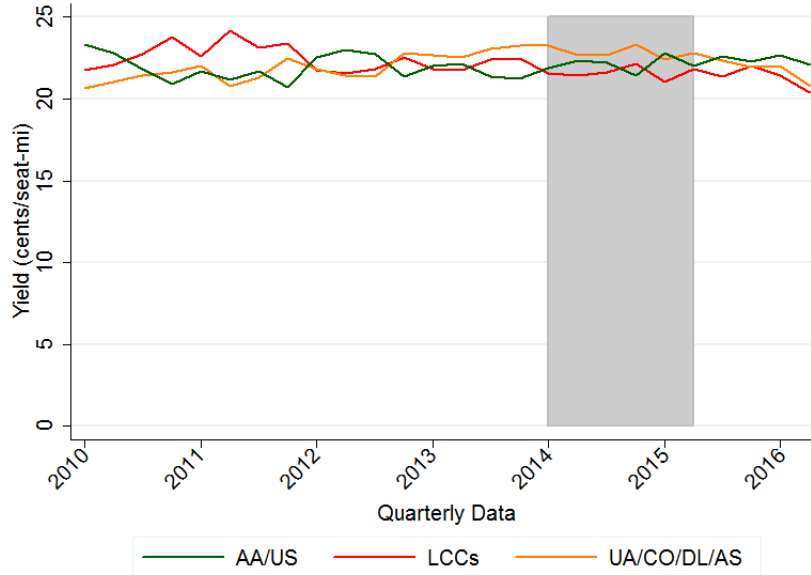
Table 2: Change in *AA/US* Yields following the merger, by route type

	Yield (¢/seat-mi)				Ln[p]
	(I)	(II)	(III)	(IV)	(V)
<i>US Airways' routes</i>					
Post-merger	0.67 (0.61)	4.86* (1.63)	2.48* (0.46)	1.22* (0.40)	0.12* (0.01)
Overall	8.19* (0.44)	4.20* (1.06)	N/A	N/A	N/A
<i>Joint routes</i>					
Post Merger	0.47 (0.50)	0.41 (1.14)	0.85* (0.37)	-0.23 (0.33)	0.05* (0.01)
Overall	-8.97* (0.30)	-6.89* (0.76)	N/A	N/A	N/A
Post-Merger Dummy	0.86† (0.45)	2.35* (0.85)	N/A	N/A	N/A
Control Variables		Y	Y	Y	Y
Route and Quarter F.E.			Y	Y	Y
Only mainline jet routes				Y	
Adj. R-Sq	0.146	0.305	0.886	0.885	0.643
N	73,831	73,831	73,831	46,067	73,831

Standard errors, in parenthesis, are clustered by quarter-route type groups.

American Airlines' routes are the omitted category. Statistically different than zero at a 5% (*) and at a 10% (†) p-value. See text for list of control variables. Mainline jet routes are those which, in 2013, AA/US utilized mainline jets, as opposed to regional jets and turbo-props, to predominantly service any leg of the route.

Figure 2: Yields on *US Airways'* routes, by Carrier Class



merging firm’s market power. However, the remedies requested by the DOJ for merger approval should have mitigated, and possibly even decreased, this market power. The largest price increase estimated, relative to the *American Airlines'* routes, is of 5% (spec. V). Other specifications estimate a price increase of 3% (specification III), or even no statistically meaningful price changes (specification II and IV). This is suggestive that the DOJ remedies were effective at restricting increases in market power due to the merger.

3.4 US Airway Prices vs Rival Prices

The above analysis showed that *AA/US* prices on *US Airways'* routes increased after the merger relative to prices on *American Airlines'* routes. However, such price changes could have resulted from prices decreasing for the American Airlines operations of the joint firm, as American Airlines was emerging from bankruptcy with renewed cost structures (e.g. new union contracts). To explore if prices did in fact increase for the US Airways operations of the joint firm, we compare post-merger changes in prices of the joint firm to those of rival firms, focusing on *US Airways'* routes.

Figure 2 shows the merging firm’s yield over time on *US Airways'* routes. It also shows the average yield of rival carriers, segmented by low-cost-carriers and rival legacy carriers, on those same routes. As with figure 1, we plot the residuals from a regression of yield on quarter and carrier class fixed effects to remove noise from seasonality and long-run carrier differences. The figure shows how US Airways/American Airlines went from being, prior to the merger, the lowest priced competitor to, post-merger, the highest priced competitor.

As before, we formalize the above analysis with a difference-in-difference estimation. Our DID

Table 3: *AA/US* Prices relative to Rivals' prices on *US Airways'* routes

	Yield (¢/pass-mi)				Ln[p]
	(I)	(II)	(III)	(IV)	(V)
AA/US * Post-merger	1.62 [†] (0.84)	2.15 (1.42)	4.54* (0.53)	1.79* (0.39)	0.10* (0.03)
AA/US	6.79* (0.61)	1.38 (0.85)	N/A	N/A	N/A
Post-Merger Dummy	-0.05 (0.73)	3.08* (0.78)	N/A	N/A	N/A
Control Variables		Y	Y	Y	Y
Route, Quarter, and Carrier F.E.			Y	Y	Y
Only mainline jet routes				Y	
Adj. R-Sq	0.04	0.20	0.778	0.804	0.620
N	54,217	54,217	54,217	26,406	54,217

Standard errors, in parenthesis, are clustered by quarter-carrier type groups. Statistically different than zero at a 5% (*) and at a 10 % (†) p-value. Sample includes only *US Airways'* routes. All rival carriers are the omitted category. See text for list of control variables. Mainline jet routes are those which, in 2013, AA/US utilized mainline jets, as opposed to regional jets and turbo-props, to service any leg of the route.

specification is:

$$yield_{irt} = \beta^M d_i^M d_t^{pst} + \gamma^M d_i^M + \gamma^{pst} d_t^{pst} + \gamma^x x_{irt}^{(2)} + \epsilon_{irt} \quad (5)$$

where d_i^M is a dummy equal to one for US Airways/American Airlines. All rival carriers form the omitted category (i.e. control group). $x_{irt}^{(2)}$ are control variables for costs and market power: a non-stop service dummy, load factor, market share on the route, the Herfindahl index, the number of carriers, non-stop carriers, low-cost carriers, and potential entrants; market share at end-point cities, and the number of non-stop routes available at end-point cities.

Table 3 shows the estimates from this DID analysis. Following the merger, yields on *US Airways'* routes increased between 1.60 and 4.50 ¢/passenger-mile more for *AA/US* than for rivals. These increases, which are statistically significant, are economically large. US Airways' average yield on these routes had been, prior to the merger, 35 ¢/passenger-mile. As such, the estimated yield increase was of at least 5%, and possibly even 13%, over and beyond what rivals' yields changed.

3.5 Price-Cost Correlations

As is clear from the past two analysis, prices of *AA/US* on *US Airways'* routes increased after the merger: they increased relative to the merging firms' prices on non-*US Airways'* routes and they increased relative to competitors' prices. The rationale for this behavior that we postulate in this paper is that marginal costs for US Airways' operations increased due to the re-optimization of fleets, in which the marginal cost of adding an additional flight changed from being that of an A320 to that of an MD80. Of course, one can also conceive alternative reasons on why prices would have

behaved as such. For example, prices could have increased if the American Airlines' frequent flyer program creates more value for customers than US Airways' program, and the merged firm adjusted prices to account for such. Similarly, if US Airways' operations inherited the American Airlines union contracts, and these union contracts increased costs for US Airways operations.

In order to show favor for the fleet re-optimization rationale over alternative hypothesis, we provide an additional empirical test. We use the Schedule P.5.2 cost information to test whether the merging firms' prices on *US Airways'* routes follow more the operating costs of the A320 over the operating costs of the MD80, and how that relationship changed after the merger. Specifically, we estimate:

$$\begin{aligned}
 yield_{rt} &= \beta^{A320,pre} c_t^{A320} d_t^{pre} + \beta^{MD80,pre} c_t^{MD80} d_t^{pre} \\
 &+ \beta^{A320,pst} c_t^{A320} d_t^{pst} + \beta^{MD80,pst} c_t^{MD80} d_t^{pst} \\
 &+ \beta^{pst} d_t^{pst} + \omega^x x_{rt}^{(3)} + \epsilon_{rt}
 \end{aligned} \tag{6}$$

where c_t^{A320} and c_t^{MD80} are the merging firms' quarterly average operating costs per passenger-seat-mile for the respective aircraft class. d_t^{pre} and d_t^{pst} are dummies for the time periods prior-to and post-merger. $x_{rt}^{(3)}$ controls for market power and costs by including a non-stop service dummy, number of carriers, number of non-stop carriers, number of low-cost carriers, potential entrants, market share at endpoint cities, the number of non-stop destinations at endpoint cities. The estimation sample includes only *US Airways'* routes and excludes routes that in 2013 did not have at least one flight segment predominantly serviced with mainline jets: Airbus A319 and larger, Boeing 737 and larger, and MD80/90.²² This excludes routes supplied exclusively by regional jets and turbo-props: e.g. Phoenix-Tucson.

The results show, presented in Table 4, how US Airways' prices prior to the merger followed more closely costs of the A320 (i.e. US Airways' work-horse aircraft) than those of the MD80 (i.e. American Airlines' work-horse aircraft). As shown in Table 4 - Specification (II), a one cent-per-passenger-mile increase in the cost of operating an A320 is correlated with a statistically significant 1.33 cent/passenger-mile increase in yield. In contrast, a similar increase in the cost of operating an MD80 is associated with a *decreases* in yield of 0.46 cents/passenger-mile. Interestingly, after the merger period, the merging firm's prices on US Airways routes are positively correlated with the MD80's cost and completely uncorrelated, statistically and economically, with the costs of the A320. Columns (III) and (IV) replicates columns (I) and (II) using a log-log specification instead of a linear-linear specification and the results are qualitatively the same: pre-merger, a one percent increase in the costs of the A320 is associated with a 0.47 percent increase in yield; a pass-through of 47 percent. Post-merger, the pass through for the A320 costs is zero, and that for the MD80 costs is of 13 percent. In summary, prior to the merger, prices on US Airways routes tracked closely the costs of the A320, but switched to tracking the costs of the MD80s post-merger.

So as to provide a falsification exercise, to test whether the above finding was not due to some

²²To be precise, a mainline jet is any jet in the Airbus's A319, A320, A321 and A330 families; in Boeing's 737, 757, and 767 families; and jets in the McDonnell Douglas MD80 and MD90 families.

Table 4: Relation Between AA/US Prices and A320 and MD80 Costs, by Route Type

	<i>US Airways' routes</i>				<i>American Airlines' routes</i>			
	Yield		Ln[p] on Ln[c]		Yield		Ln[p] on Ln[c]	
	(I)	(II)	(III)	(IV)	(V)	(VI)	(VII)	(VIII)
A320 x	1.64*	1.33*	0.62*	0.47*	0.07	-0.09	0.20*	-0.01
Pre-merger	(0.33)	(0.10)	(0.05)	(0.03)	(0.18)	(0.16)	(0.04)	(0.05)
MD80 x	-0.73*	-0.46*	-0.27*	-0.18*	0.33*	0.43*	0.05	0.19*
Pre-merger	(0.22)	(0.07)	(0.05)	(0.03)	(0.12)	(0.09)	(0.03)	(0.04)
A320 x	1.32*	0.04	0.03	0.03	-0.80*	-0.84*	-0.18*	-0.20*
Post-merger	(0.44)	(0.13)	(0.05)	(0.03)	(0.25)	(0.10)	(0.04)	(0.03)
MD80 x	4.01*	0.54*	0.10	0.13†	0.80†	0.48*	0.12	0.13*
Post-merger	(0.86)	(0.25)	(0.14)	(0.07)	(0.49)	(0.20)	(0.09)	(0.07)
Post-merger	-36.1*	2.42	0.47	0.29†	2.33	6.58*	0.67*	0.59*
	(8.35)	(2.48)	(0.32)	(0.17)	(4.79)	(2.07)	(0.22)	(0.16)
Controls		Y		Y		Y		Y
Route F.E.		Y		Y		Y		Y
Adj. R-Sq	0.006	0.918	0.015	0.764	0.002	0.827	0.014	0.487
N	12,140	12,140	12,140	12,140	21,220	21,220	21,220	21,220

Standard errors in parenthesis. Statistically different than zero at a 5% (*) and at a 10 % (†)

p-value. Sample excludes the five quarters of the merger period and the routes which did not have at least one flight segment predominantly served with mainline jets in 2013. Yield and costs are measured in ¢/passenger-mile. Ln[p] on Ln[c] regress log of average price (\$/passenger) on log of aircraft operating costs. See text for list of control variables.

spurious correlation between the costs of the different aircraft and prices on *US Airways'* routes, we repeat the exact same analysis except that we change the estimation sample to be prices of *AA/US* on *American Airlines'* routes.²³ The results, also shown in Table 4, show that on *American Airlines'* routes, *AA/US* prices follow more closely the costs of the MD80, both before and after the merger, than the costs of the A320. We interpret this as evidence that the merging firms' prices post-merger respond to the merging firms' marginal costs: those of the more expensive MD80.

The costs of the MD80 and of the A320 vary over time, but not across routes. Hence, the above regressions can be subject to col-linearity issues, as there are three variables that are identified off the five quarters post-merger (2015Q2-2016Q2): *MD80 x Post-merger*, *A320 x Post-merger*, and *Post-merger*. To address this concern we build four different cost variables, and have non-nested tests dictate which of these four cost variables best fit the pricing data. The first two cost variables are simply the cost of the A320 over time and the cost of the MD80 over time: c_t^{A320} , c_t^{MD80} . The third cost variable follows the cost of the A320 up to the merger, and that of the MD80 after the merger. The fourth does the opposite:

$$c_t^{A320/MD80} = c_t^{A320} d_t^{pre} + c_t^{MD80} d_t^{pst} \quad (7)$$

$$c_t^{A320/MD80} = c_t^{MD80} d_t^{pre} + c_t^{A320} d_t^{pst} \quad (8)$$

For each one of these variables, and for each of the two route types studied (i.e. *US Airways'* routes and *American Airlines'* routes), we run the OLS regression:

$$yield_{rt} = \beta^v c_t^v + \beta^{pst} d_t^{pst} + \varphi^x x_{rt}^{(3)} + \epsilon_{rt} \quad (9)$$

where c_t^v is one of the four cost variables. $x_{rt}^{(3)}$ contains the same controls as in Table 4: a dummy for non-stop service, number of carriers, number of non-stop carriers, number of low-cost carriers, potential entrants, market share at endpoint cities, the number of non-stop destinations at endpoint cities, a dummy for American Airlines during their bankruptcy period, and route fixed effects.

Using the regression results, we test whether the model with $c_t^{A320/MD80}$ outperforms the other models in terms of fit. Specifically, we use Vuong (1989)'s likelihood ratio test for non-nested models. As the tests assume log-likelihoods, we assume the error term is distributed normal and iid and calculate the corresponding log-likelihoods. We also test if the model with $c_t^{MD80/MD80}$ outperforms the other models. Results are presented in table 5.

As suggested, when fitting yields on *US Airways'* routes, the model in which costs are those of the A320 prior to the merger and those of the MD80 after the merger outperforms all other models. It is the model with the highest R-sq and likelihood ratio test confirms that it outperforms all other models. Interestingly, when fitting yields on *American Airlines'* routes the model in which costs are those of the MD80 both before and after the merger is the one which best fits that data.

²³We also add a dummy variable for the bankruptcy period.

Table 5: Regression of *US/AA* Yields on Alternative Cost Variables and Corresponding Likelihood Ratio Tests

Cost Variable [Pre / Post]	<i>US Airways'</i> routes				<i>American Airlines'</i> routes			
	(I)	(II)	(III)	(IV)	(I)	(II)	(III)	(IV)
	A320 / MD80	A320 / A320	MD80 / MD80	MD80 / A320	A320 / MD80	A320 / A320	MD80 / MD80	MD80 / A320
Estimate	0.78*	0.62*	0.20*	0.17*	0.62*	0.22*	0.39*	0.26*
Std. Error	(0.06)	(0.06)	(0.04)	(0.04)	(0.08)	(0.05)	(0.03)	(0.03)
Adj. R-sq	0.9181	0.9178	0.9172	0.9172	0.8262	0.8254	0.8264	0.8258
Log-LH	-33,968	-33,985	-34,031	-34,033	-60,624	-60,671	-60,612	-60,648
LR-test: model (I) outperforms column model?								
Z-statistic	N/A	2.51	6.70	6.39	N/A	5.97	-2.44	3.88
P-value		0.994	0.999	0.999		0.999	0.007	0.999
LR-test: model (III) outperforms column model?								
Z-statistic	-6.70	-5.50	N/A	1.29	2.44	6.35	N/A	6.83
P-value	0.000	0.000		0.901	0.993	0.999		0.999

Sample includes only those routes who, in 2013, AA/US predominantly serviced at least one flight segment with mainline jets: Airbus A-319/320/321/330, Boeing 737/757/767 and MD-80/90. Sample excludes the five quarters of the merger prior: 2014Q1-2015Q1. See text for list of control variables.

4 Discussion

These tests confirm our hypothesis that the merged firm's prices on US Airways routes increased after the merger, mostly due to the reallocation of aircraft, in which the *marginal* aircraft for US Airways' operations became the MD80. The price increases are large, between five and fifteen percent, and consumers are likely to have been affected. A formal estimate of the potential consumer welfare loss is beyond the scope of this paper.²⁴ However, the price increases estimated are significantly above the FTC's suggested five percent increase for SSNIP tests. We do not suggest that welfare losses are as large as suggested by our (extremely) simplified model of competition in section 2.2, as consumers may have responded to the price increase by switching to more affordable rival carriers, and not by leaving the industry as the simplified model suggests. More importantly, American Airlines has initiated a massive fleet renewal project in an effort to retire its MD80 fleet. As American Airlines renews its fleet and its marginal aircraft becomes either a Boeing-737 or an Airbus A320, we should expect prices to decrease, potentially to pre-merger levels.

The purpose of this exercise was to show how production rationalization across merging parties may result in lower average costs for the merging parties but in higher marginal costs for a subset of the merging firms. Many proposed mergers present, to both investors and regulators, production reallocation synergies as one of the key benefits of the merger. We caution both investors and regulators

²⁴To obtain formal estimates on changes in consumer surplus one needs, at a minimum, a model of demand, and, preferably, a model of supply, so as to include how rivals' react to the merged firm's pricing strategies. These models involve complex data processing and, more importantly, strong modeling and identifying assumptions. As welfare calculations are susceptible to such assumptions, we prefer to leave that exercise to future research so as to give the exercise the proper attention it requires.

that while such production re-allocation can decrease average cost, it can increase marginal cost and this increase in marginal cost may place the merging firm at a disadvantage relative to rivals, resulting in higher prices and lower sales.

The merger effects proposed here are a subset of cross-market dynamics that have become common as firms globalize. For example, over the past three decades global cement manufacturing has become concentrated, with global manufacturers shipping production across continents in an effort to hedge demand and supply swings. It is unfortunate that the FTC's Horizontal Merger Guidelines state so little about the benefits and harms that may come to consumers from such cross-market dynamics, including diseconomies of scope (Bulow et al. (1985)), multi-market contact (Bernheim and Whinston (1990), Arie et al. (2016)), and the effects presented here.

5 Conclusion

This paper contains a simple idea: production rationalization averages cost curves across facilities, decreasing costs in some markets and increasing costs in others. Mergers whose efficiencies (a.k.a. synergies) are based on such rationalizations may be beneficial to the merging parties but detrimental to consumers, even if there are no shifts in market power.

The US Airways/American Airlines merger provides a great example of how such rationalization can negatively affect consumers while positively affecting the merging parties. As US Airways operated more efficient aircraft than American Airlines prior to their merger, and US Airways had slack in the utilization of such aircraft, the merger allowed the merging parties to fully utilize such efficient aircraft but altered US Airways' marginal cost from a low-cost aircraft (the A320) to a high-cost aircraft (the MD80). Our empirical analysis shows that this resulted in prices of the merging firm rising 10% more on US Airways' routes than on American Airlines' routes and 5% more than rivals' prices on US Airways' routes. Such price hikes are higher than the 5% threshold commonly utilized by the antitrust agencies in determining potentially anti-competitive mergers.

When promoting mergers, 'synergies' tend to be highly invoked but much less understood. This paper takes on one such synergy, production re-allocation, and illustrates how it affects both firm profits and consumer welfare. The paper pin-points how such synergy could have come about in the US Airways/American Airlines merger and the effect it had on prices. It would be of great value to businessmen and regulators alike to see clear examples of other synergies at play, including increasing economies of scale, know-how diffusion, and information benefits.

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A Proof of Propositions

Before providing a proof for the main proposition in the text, we introduce a few auxiliary functions and provide context on them.

Definition. Define $F^i : \mathbb{R}_+^4 \rightarrow \mathbb{R}^2$, $\hat{\varphi}_i : \mathbb{R}^2 \rightarrow \mathbb{R}$ and $\hat{\mu}_i : \mathbb{R}^2 \rightarrow \mathbb{R}$, where $F^i(\varphi, \tau, \mu, Q_{-i}) \equiv \{\pi_\varphi^i + \mu, \mu\varphi\}$ and $\hat{\varphi}_i(\tau, Q_{-i})$ and $\hat{\mu}_i(\tau, Q_{-i})$ are implicitly derived from $F^i(\varphi, \tau, \mu, Q_{-i}) = 0$.

Claim 1. $\hat{\varphi}_i(\tau, Q_{-i})$ and $\hat{\mu}_i(\tau, Q_{-i})$ exist, are continuous, and their first derivatives exist.

Proof. (omitting the i scripts) $\hat{\varphi}(\tau, Q_{-i})$ and $\hat{\mu}(\tau, Q_{-i})$ exist if the matrix $\begin{bmatrix} \pi_{\varphi\varphi} & 1 \\ \mu & \varphi \end{bmatrix}$ has an inverse at values that satisfy $F(\varphi, \tau, \mu, Q_{-i}) = 0$ and $(\varphi, \mu, \tau, Q_{-i}) \geq 0$. As the matrix is two-by-two, existence of an inverse is equivalent to its determinant being non-zero. That is, that $\varphi\pi_{\varphi\varphi} - \mu \neq 0$. This is always the case: if $\varphi > 0$, then $\pi_{\varphi\varphi} = R_{qq}\vartheta_\varphi^2 + R_q\vartheta_{\varphi\varphi} - C_{\varphi\varphi} \leq R_{qq}\vartheta_\varphi^2 < 0$ (by assumption XX) and since $\mu \geq 0$, the determinant is strictly negative. If $\varphi = 0$, then $\pi_\varphi = R_q\vartheta_\varphi - C_\varphi = -C_\varphi < 0$ (by assumption XX) and hence $\mu \neq 0$.

These derivatives are given by the implicit function theorem. As the derivatives exist, the functions are continuous. \square

Claim 2. For values of τ and Q_{-i} such that $\hat{\mu}_i(\tau, Q_{-i}) = 0$, $R_q^i(\vartheta^i(\tau, \hat{\varphi}_i(\tau, Q_{-i})), Q_{-i}) > 0$.

Proof. If $\hat{\mu}_i = 0$ then τ, Q_{-i} and $\hat{\varphi}_i(\tau, Q_{-i})$ are such that $\pi_\varphi^i = 0 = R_q^i\vartheta_\varphi - C_\varphi^i$. As $C_\varphi^i > 0$ and $\vartheta_\varphi \geq 0$, it follows that $R_q^i > 0$. \square

Claim 3. $\frac{d}{d\tau}\vartheta^i(\tau, \hat{\varphi}_i(\tau))$ is increasing in τ .

Proof. Calculating the derivative, and omitting the i -scripts, $\frac{d}{d\tau}\vartheta = \vartheta_\tau + \vartheta_\varphi \frac{d\varphi}{d\tau}$. By the implicit function theorem,

$$\frac{d\hat{\varphi}}{d\tau} \equiv \begin{cases} -\pi_{\varphi\tau}/\pi_{\varphi\varphi} & \text{if } \hat{\mu}(\tau, Q_{-i}) = 0 \\ 0 & \text{otherwise} \end{cases}$$

Hence, if $\hat{\mu}(\tau, Q_{-i}) \neq 0$, $\frac{d}{d\tau}\vartheta = \vartheta_\tau > 0$. If $\hat{\mu}(\tau, Q_{-i}) = 0$, then

$$\begin{aligned} \frac{d\vartheta}{d\tau} &= \vartheta_\tau - \vartheta_\varphi \frac{\pi_{\varphi\tau}}{\pi_{\varphi\varphi}} = \frac{1}{\pi_{\varphi\varphi}} \{ \vartheta_\tau \pi_{\varphi\varphi} - \vartheta_\varphi \pi_{\varphi\tau} \} \\ &\geq \frac{1}{\pi_{\varphi\varphi}} \{ \vartheta_\tau (R_{qq}\vartheta_\varphi^2 + R_q\vartheta_{\varphi\varphi}) - \vartheta_\varphi (R_{qq}\vartheta_\tau\vartheta_\varphi + R_q\vartheta_{\varphi\tau}) \} \\ &= \frac{R_q}{\pi_{\varphi\varphi}} \{ \vartheta_\tau\vartheta_{\varphi\varphi} - \vartheta_\varphi\vartheta_{\varphi\tau} \} \end{aligned}$$

where the inequality follows from $R_{qq}\vartheta_\varphi^2 + R_q\vartheta_{\varphi\varphi} \leq 0$ and $C_{\varphi\varphi} \geq 0$. As $R_q > 0$, $\pi_{\varphi\varphi} < 0$, it is enough to show the term in curly brackets is positive. It is, as $(\vartheta_\tau, \vartheta_\varphi, \vartheta_{\varphi\tau}) \geq 0$ and $\vartheta_{\varphi\varphi} \leq 0$. \square

For clarity, we re-state the proposition of the main text and develop a formal proof.

Proposition. *If, without production rationalization, firm a's equilibrium marginal cost is higher than firm b's, then with production rationalization:*

1. *Firm a's sales increase and firm b's decrease.*
2. *Total sales in market A increases and total sales in market B decrease.*
3. *Consumer surplus increases in market A and decreases in market B.*

The opposite effects occur if, without production rationalization, firm a's equilibrium marginal cost is lower than firm b's.

Proof. Denote as a *rival firm* any firm that is neither firm *a* nor firm *b*. We first establish that rival firms' problem can be recast to one in which they choose sales (i.e. q) instead of inputs, as can firm *a* and *b*'s problem in the absence of production rationalization. Then we show existence of equilibrium when production rationalization is infeasible. Third, we show equilibrium with production rationalization can be characterized by ...

Recast *rival firms'* problem as a single variable problem:

$$\max_{\tau \geq 0} R^i(\vartheta^i(\tau, \hat{\varphi}_i(\tau, Q_{-i}^*)), Q_{-i}^*) - C^i(\tau, \hat{\varphi}_i(\tau, Q_{-i}^*))$$

It is clear from the definition of $\hat{\varphi}_i(\tau, Q_{-i})$ that this new problem is identical to the original problem. As $\vartheta^i(\tau, \hat{\varphi}_i(\tau, Q_{-i}))$ is strictly increasing in τ , define $\hat{\tau}_i(q, Q_{-i})$ as the inverse function to $q = \vartheta^i(\tau, \hat{\varphi}_i(\tau, Q_{-i}^*))$. Also, for ease of notation, let $c^i(q, Q_{-i}) \equiv C^i(\hat{\tau}_i(q, Q_{-i}^*), \hat{\varphi}_i(\hat{\tau}_i(q, Q_{-i}^*), Q_{-i}^*))$. *Rival firms'* problem can now be written as

$$q_i^* = \arg \max_{q \geq 0} R^i(q, Q_{-i}^*) - c^i(q, Q_{-i}^*)$$

When production rationalization is infeasible, a similar transformation can be done with firm *a* and *b*'s problem. As such, the game resembles a Cournot game. Existence of equilibrium follows from XXXX (Vives pg. YYY).

The rest of the proof is as follows:

1. Show that, with production rationalization, *rival firms'* best response functions are such that *rival firms'* aggregate sales are decreasing in firm *a*'s and *b*'s sales, industry sales are increasing in *a* and *b*'s sales, and this aggregate best response, along with firm *a* and *b*'s FOCs, are sufficient to characterize the equilibrium.
2. Show that firm *a*'s and *b*'s marginal revenue in a given market is decreasing in the rationalizing firm's output in that market, taking into account *rival firms'* aggregate best response.

3. Show that, for any choice of τ , there are corresponding optimal inputs φ (i.e. $\varphi_a^*(\tau)$ and $\varphi_b^*(\tau)$) and that output ϑ is increasing in input τ after considering the optimal $\varphi_m^*(\tau)$: $\frac{d}{d\tau}\vartheta^m(\tau, \varphi_m^*(\tau)) > 0$. Hence, there is a one-to-one mapping between choice of input τ and sales, after considering optimal input φ .
4. Show that a 's equilibrium marginal revenue in market A is (weakly) lower with production rationalization than without it and that the opposite occurs for b in market B .

Given no. 4, as equilibrium marginal revenue is decreasing in a and b 's output (no. 2 above), the first statement of the proposition follows. The second line of the proposition then follows from no. 1 above. Finally, as consumer surplus is increasing in total industry output, the last statement of the proposition follows.

[[[NOT FINISHED]]] [[[PROOF BELOW IS INCOMPLETE / NEEDS WORK]]]

First, for any rival firm i :

$$R_q^i - c_q^i = 0$$

By the inverse function theorem,

$$\frac{dq_i^*}{dQ_{-i}} = -\frac{R_{qQ}^i - c_{qQ}^i}{R_{qq}^i - c_{qq}^i} >? \in (-1, 0)$$

The aggregate response of all firms, but firm a , for firm a 's output is

$$\sum_{i \neq a} \frac{dq_i}{dq_a} = \sum_{i \neq a} \frac{dq_i}{dQ_{-i}} \frac{dQ_{-i}}{dq_a}$$

and since $\frac{dQ_{-i}}{dq_a} \geq 0$ and $\sum_{i \neq a} \frac{dQ_{-i}}{dq_a} \leq 1$ by definition of Q_{-i} , then $\sum_{i \neq a} \frac{dq_i}{dq_a} \in (-1, 0)$. This shows aggregate rivals' best response to firm a 's output is decreasing in firm a 's output and equilibrium industry output is increasing in a 's equilibrium output.

Second, let $Q_{-a}(q_a)$ be the aggregate output derived from inverting rival firms' (not firm a) FOC with respect to firm a 's output. That is, $Q_{-a}(q_a)$ is the inverse function derived from the system of equations:

$$R_q(q_i, q_a + \sum_{j \neq i, a} q_j) - C'_i(q_i) = 0 \quad \forall i \neq a \quad ; \quad Q_{-a} = \sum_{i \neq a} q_i$$

Recall that as a and b 's rivals' FOCs are identical when production rationalization is feasible for a and b as when it is not, the function $Q_{-a}(\cdot)$ is the same for both scenarios. Also note that as demand is concave, a 's (and b 's) marginal revenue is decreasing in a 's (in b 's) output after considering rivals' equilibrium response:

$$\frac{d}{dq_a} R_q^a(q_a, Q_{-a}(q_a)) = P'_A(Q) + (q_i P''_A(Q) + P'_A(Q)) \left(\frac{d}{dq_a} (q_a + Q_{-a}(q_a)) \right) < 0$$

Third, so as to ease notation, denote equilibrium marginal revenue and marginal cost values of firms a and b , when production rationalization is feasible, as:

$$\eta_a^* \equiv \eta_A(q_a^*, Q_{-a}(q_a^*)) \quad \eta_b^* \equiv \eta_B(q_b^*, Q_{-b}(q_b^*)) \quad c_a^* \equiv C'(q_a^*) \quad c_b^* \equiv C'(q_b^*)$$

where (q_a^*, q_b^*) are the equilibrium sales when production rationalization is feasible. It also adds clarity if we expand the rationing firms' actions to include production amounts in each market's facility, $(\vartheta_a^*, \vartheta_b^*)$, and add to the firms' joint problem a sales-to-production constraint: $\vartheta_a^* + \vartheta_b^* \geq q_a^* + q_b^*$. These four decision variables relate to the original problem's decision variables as: $q_a^* \equiv q_a + \vartheta$, $q_b^* \equiv q_b$, $\vartheta_a^* \equiv q_a$, $\vartheta_b^* \equiv q_b + \vartheta$. The firms' joint problem is then:

$$\begin{aligned} \max_{q_a, q_b, \vartheta_a, \vartheta_b} \quad & P_A(\hat{q}_a + Q_{-aA}^* \hat{q}_a) + P_B(\hat{q}_b + Q_{-bB}^* \hat{q}_b) - C_a(\vartheta_a) - C_b(\vartheta_b) \\ \text{s.t.} \quad & (q_a, q_b, \vartheta_a, \vartheta_b) \geq 0 \quad q_a + q_b \leq \vartheta_a + \vartheta_b \end{aligned}$$

The equilibrium, given this problem and rivals' problems, is thus characterized by the equations:

$$\begin{aligned} \eta_a^* - \bar{\mu} + \mu_{q_a}^* &= 0 & \eta_b^* - \bar{\mu} + \mu_{q_b}^* &= 0 & -c_a^* + \bar{\mu} + \mu_{\vartheta_a}^* &= 0 & -c_b^* + \bar{\mu} + \mu_{\vartheta_b}^* &= 0 \\ \bar{\mu}(\vartheta_a^* + \vartheta_b^* - q_a^* - q_b^*) &\geq 0 & \bar{\mu} &\geq 0 & \mu_{q_a}^* q_a^* &= 0 & \mu_{q_b}^* q_b^* &= 0 & \mu_{\vartheta_a}^* \vartheta_a^* &= 0 & \mu_{\vartheta_b}^* \vartheta_b^* &= 0 \end{aligned}$$

where $(\mu_{q_a}^*, \mu_{q_b}^*, \mu_{\vartheta_a}^*, \mu_{\vartheta_b}^*)$ are the Lagrangian multipliers on the non-negativity constraints of the respective choice variables and $\bar{\mu}$ is the Lagrangian multiplier on the constraint regarding sales to production.

Similarly, denote the equilibrium values when production rationalization is not allowed as

$$\hat{\eta}_a \equiv \eta_A(\hat{q}_a, Q_{-a}(\hat{q}_a)) \quad \hat{\eta}_b \equiv \eta_B(\hat{q}_b, Q_{-b}(\hat{q}_b)) \quad \hat{c}_a \equiv C'(\hat{q}_a) \quad \hat{c}_b \equiv C'(\hat{q}_b)$$

where (\hat{q}_a, \hat{q}_b) are the equilibrium sales when production rationalization is feasible. These equilibrium quantities are characterized by the equations:

$$\begin{aligned} \hat{\eta}_a - \hat{\mu}_{q_a} &= 0 & \hat{\eta}_b + \hat{\mu}_{q_b} &= 0 & -\hat{c}_a + \hat{\mu}_{q_a} &= 0 & -\hat{c}_b + \hat{\mu}_{q_b} &= 0 \\ \hat{\mu}_{q_a} \hat{q}_a &= 0 & \hat{\mu}_{q_b} \hat{q}_b &= 0 \end{aligned}$$

And because of assumption 1, which states firms a and b are active in their respective markets without production rationalization, the above equations (without rationalization) can be reduced to $\hat{\eta}_a = \hat{c}_a$ and $\hat{\eta}_b = \hat{c}_b$.

The premise of the proposition states $\hat{c}_a > \hat{c}_b$ which implies $\hat{\eta}_a > \hat{\eta}_b$. Notice the following for the case with production rationalization :

- By the equilibrium equations, if equilibrium production at both facilities is positive (i.e.

$(\vartheta_a^*, \vartheta_b^*) > 0$), marginal costs must be equal: $c_a^* = c_b^*$. If production at one facility is zero (e.g. $\vartheta_a^* = 0$), marginal cost at that facility must be larger than at the other facility (i.e. $c_a^* \geq c_b^*$).

- Similarly, by the same equations, if equilibrium sales in both markets are positive (i.e. $(q_a^*, q_b^*) > 0$), marginal revenue must be equal across markets: $\eta_a^* = \eta_b^*$. If sales in one market are zero (e.g. $q_a^* = 0$), marginal revenue in that market must be smaller than in the other market (e.g. $\eta_a^* < \eta_b^*$).
- Finally, by the same equations, marginal revenue of markets where sales take place is equal to marginal cost of facilities where production takes place, and, by the above statements, $\min\{\eta_a^*, \eta_b^*\} \leq \max\{c_a^*, c_b^*\}$.

We now show $\eta_b^* \geq \hat{\eta}_b$ and, by consequence, $q_b^* \geq \hat{q}_b$. Assume $\eta_b^* < \hat{\eta}_b$ such that $q_b^* > \hat{q}_b$. By bullet-point no. 2 above, it must be that $\eta_a^* \leq \eta_b^*$, which implies $\eta_a^* < \hat{\eta}_a$. This in turn implies that $q_a^* > \hat{q}_a$, sales in both markets are positive with $\eta_a^* = \eta_b^*$, and $q_a^* + q_b^* > \hat{q}_a + \hat{q}_b$. As cost curves are weakly convex, and $\hat{c}_b < \hat{c}_a$, there must be positive production in market B with $c_b^* \geq \hat{c}_b$. Thus, the equilibrium equations include $\eta_a^* = \eta_b^* = c_b^*$ (i.e. bullet-points number 1 and 3 above). But this contradicts the previous statements: $\eta_b^* < \hat{\eta}_b = \hat{c}_b \leq c_b^* = \eta_b^*$, a contradiction.

The case for $\eta_a^* \leq \hat{\eta}_a$ follows similar arguments. Assume $\eta_a^* > \hat{\eta}_a$, which implies $q_a^* < \hat{q}_a$. $\eta_b^* \geq \eta_a^* > \hat{\eta}_a > \hat{\eta}_b$ implies $q_b^* < \hat{q}_b$ and thus $q_a^* + q_b^* < \hat{q}_a + \hat{q}_b$. If $\bar{\mu} \neq 0$ such that production facilities do not produce spare output (i.e. the sales to production constraint holds strict), then it must be that $\max\{c_a^*, c_b^*\} \leq \hat{c}_a$. However, since the equilibrium without production rationalization has $\hat{c}_a = \hat{\eta}_a$, and with production rationalization $\eta_a^* \leq \min\{\eta_a^*, \eta_b^*\} \leq \max\{c_a^*, c_b^*\}$, thus $\eta_a^* \leq \hat{c}_a = \hat{\eta}_a < \eta_a^*$, a contradiction. If, on the other hand, $\bar{\mu} = 0$, by the equations above $\min\{\eta_a^*, \eta_b^*\} = 0$, which contradicts $\eta_a^* > \hat{\eta}_a > 0$ and $\eta_b^* > \hat{\eta}_b > 0$.

In summary, $\eta_a^* \leq \hat{\eta}_a$ and $\eta_b^* \geq \hat{\eta}_b$ imply $q_a^* \geq \hat{q}_a$ and $q_b^* \leq \hat{q}_b$, which in turn implies $q_a^* + Q_{-a}(q_a^*) \geq \hat{q}_a + Q_{-a}(\hat{q}_a)$ and $q_b^* + Q_{-b}(q_b^*) \leq \hat{q}_b + Q_{-b}(\hat{q}_b)$. \square